Lattice-Enhanced Screening Parameterization

Equation Derivation and Test Procedure

1. Model Derivation

This model is a phenomenological, unit-consistent formulation describing how lattice coherence and loading modify barrier penetration probabilities through an effective screening energy U_e. No first-principles nuclear assumptions are made; all enhancements are introduced as bounded, dimensionless modifiers applied to a baseline screening energy.

Define two measurable, dimensionless observables: coherence C(t) and loading $\theta(t)$, both constrained to the interval [0,1]. These observables modify the baseline screening energy U0 multiplicatively:

$U_e(C, \theta) = U0 \cdot (1 + \alpha C) \cdot (1 + \beta\theta)$, where $U0 \geq 0$ and $\alpha, \beta \geq 0$.

This preserves energy units while ensuring bounded behavior. The effective collision energy is shifted to E + U_e inside the tunneling exponential and prefactor of the standard astrophysical S-factor cross-section.

The screened cross-section is written as $\sigma_eff(E;C,\theta) = [S(E+U_e)/(E+U_e)]\cdot\exp(-2\pi\cdot\eta(E+U_e))$, where η is the Sommerfeld parameter expressed in energy form and remains dimensionless.

Ensemble averaging over an energy probability density p(E), normalized to unity, produces the effective reactivity ■σv■. Multiplication by particle densities yields the reaction rate density R, which integrates over volume to give nuclear power via multiplication by the reaction Q-value.

2. Energy Conservation

Energy accounting is explicit. The screening term U_e modifies reaction probability only and does not contribute energy. Any energy required to create or maintain the screened state must appear in P_in or P_stored. Nuclear power is credited solely through the reaction Q-value.

3. Numerical Test Procedure

1. Specify physical constants, particle species, reduced mass μ, charges Z1 and Z2, and reaction Q-value.
2. Choose a normalized energy distribution p(E) (e.g., Maxwellian at temperature T).
3. Set model parameters U0, α, β and define operational inputs C and θ within [0,1].
4. Compute the Sommerfeld parameter $\eta(E+U_e)$ using consistent energy units.
5. Evaluate $\sigma_eff(E;C,\theta)$, v(E), and numerically integrate ■σv■ over energy.
6. Verify convergence by extending energy bounds and refining the integration grid.
7. Compute R = n1·n2·■σv■ and P_nuc = Q·R·V.
8. Confirm energy closure by ensuring P_out – P_in is explained by P_nuc and explicit storage or loss terms.

This procedure is sufficient to verify dimensional consistency, boundedness, convergence, and energy conservation. Agreement or disagreement with experimental data determines physical validity.